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Novel Method of Friction Identification with Application for Inverted Pendulum Control System

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This text covers a novel method of friction identification for control systems. The friction function in the inverted pendulum model is described by means of a cubic polynomial. The method has been tested using the data recorded on a real inverted pendulum. It has been proven that the proposed cubic model offers the same level of accuracy as the Coulomb model. However, all the difficulties caused by Coulomb's model discontinuity are omitted.

Keywords: Friction, identification, Coulomb model, cubic model, inverted pendulum, control.

1. Introduction

This paper is focused on friction identification for the purposes of control systems analysis. In order to reliably predict behavior of a control system, a mathematical model of the device under consideration must be created. The model is expected to account for all the phenomena that take place in the system which are important from the point of view of control. Moreover, the model is usually dependent on a set of parameters, whose values have to be selected. The process of selection of model parameters in such a way that the behavior of the model is as close as possible to behavior of the real device is called an identification.

Typically, nonlinear continuous-time control systems are described by means of differential equations. To make sure that the equations model dynamics of the system with sufficient accuracy, all the parameters must be estimated on the basis of experimental data [1]. An important issue in identification of mechanical control objects is friction modelling. Different friction models are available [2]. They differ by complexity, range of applications etc.

In this paper a novel approach is shown: the friction function has been represented by means of a simple, two-parameters cubic model. The new method is applied to a model of an inverted pendulum. It has been experimentally proven that, in the proposed range of applications, the cubic model can be as accurate as the classical Coulomb model. Moreover, it helps to avoid all the problems caused by Coulomb model's discontinuity, such as inability to linearize or difficulties in stability investigations.

2. Modelling of the Control Object

The control object analyzed in this paper is an inverted pendulum (Fig. 1). The inverted pendulum is a kind of pendulum in which the axis of rotation is fixed to a cart. The cart is able to move along the horizontal axis x in a controlled way. The fundamental problem of the inverted pendulum is to find such a control of the cart that keeps the pendulum's bar in the vicinity of the upright vertical position $\alpha(t) = 0$ even if external disturbances appear.

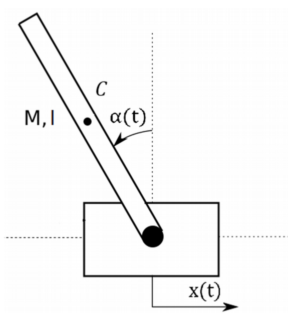

Figure 1 Sketch of the considered control object – the inverted pendulum

It has been assumed that the pendulum's drive is velocity-controlled. It means that the control signal $u(t)$ supplied to the drive is equal to the desired velocity of the cart. If the drive is stiff enough, then the motion of the pendulum's bar does not influence position of the cart $x(t)$. Providing that the drive can be approximated by a linear differential equation of the first order, the dependence between acceleration of the cart and the control signal is as follows (1):

$$\ddot{x}(t) = a[u(t) - \dot{x}(t)] \tag{1}$$

where $u(t)$ is the control signal and a is a drive constant, which can be determined in the identification process.

The equation of motion of the inverted pendulum can be easily derived using Lagrange approach [3]. Assume that the pendulum's bar is uniform, its mass center C is in the middle of its length and it is loaded by a friction torque $\tau * ml^2/3$. Then, the following equation of motion (2) is obtained:

$$\ddot{\alpha}(t) = \frac{3g}{2l}\sin(\alpha(t)) + \frac{3\ddot{x}(t)}{2l}\cos(\alpha(t)) - \tau(\dot{\alpha}(t)) \tag{2}$$

where l is the length of the bar. Equations (1) and (2) constitute a complete mathematical description of the inverted pendulum.

It can be noticed that, as long as the drive can be described by a linear differential equation of the first order, building a mathematical model of the inverted pendulum is straightforward. However, the equation (2) contains one part whose modelling is not trivial – the friction function $\tau()$.

Direct identification of the dependency between the angular velocity of the bar and the friction function $\tau()$ requires a precise measurement of the torque for different rotational velocities of the pendulum bar's shaft. Such method requires costly equipment. Therefore, a different approach must be found.

It seems that one of the simplest methods to investigate the friction function is to analyze free vibrations of the pendulum when the position of the cart is fixed. A scheme of measurement is presented in the Fig. 2.

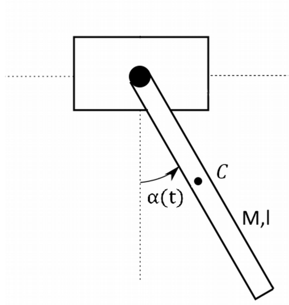

Figure 2 Measurement of pendulum's bar free vibrations

Free vibrations of the pendulum are described by the differential equation (3):

$$\ddot{\alpha}(t) = -\frac{3g}{2l}\sin(\alpha(t)) - \tau(\dot{\alpha}(t)) \tag{3}$$

Registration of free vibrations of the pendulum results in values of the function $\alpha(t)$ in some discrete moments of time $t = 0$, $t = T, \ldots, t = kT$, where T is the sampling period and k is the number of samples. From the obtained data, initial conditions of the motion can be estimated. Assume that a proposition of friction model $\tau^*()$, which depends on a vector of parameters $\mathbf{c}= [c_1, c_2, \ldots, c_m]$, is selected. Then, the equation (3) can be simulated numerically. Let the initial conditions of the simulation be approximately the same as in the registered motion of the real pendulum. In such case, the simulation yields an approximate function $\alpha^*(t)$. If the friction model is selected correctly and values of the parameters vector $\mathbf{c}$ are properly adjusted, then the discrepancy between $\alpha(t)$ and $\alpha^*(t)$ should be small.

According to [4], the standard error of estimation is defined as (4):

$$\sigma = \sqrt{\frac{\sum_{i=0}^{k}[\alpha(iT) - \alpha^*(iT)]^2}{k+1}} \tag{4}$$

If the initial conditions of motion are found with sufficient accuracy, then the standard error σ is a function of model parameters: $\sigma = \sigma(\mathbf{c})$. This function can be minimized by means of an optimization procedure. Such approach is referred as *Data fitting by numerical approximation of an initial value problem* [1].

3. Identification of the Friction Function

There exist a large variety of available models that describe friction in multibody systems [2]. Their structures differ depending on a desired application. In the case of pendulum's friction model, effects that can be observed for very small relative velocities (i.e., stick-slip behavior or Stribeck effect) are not crucial, because the pendulum in free vibrations mostly rotates with significant rotational velocities. Therefore, two standard friction models are investigated: a linear model (5) and a Coulomb model (6). The novel approach introduced in this paper is application of the cubic model (7).

$$\tau(\dot{\alpha}) = c_1\dot{\alpha} \tag{5}$$

$$\tau(\dot{\alpha}) = c_1\dot{\alpha} + c_2 sgn(\dot{\alpha}) \tag{6}$$

$$\tau(\dot{\alpha}) = c_1\dot{\alpha} + c_2(\dot{\alpha})^3 \tag{7}$$

An exemplary motion, recorded on a real pendulum, is depicted in the Fig. 3.

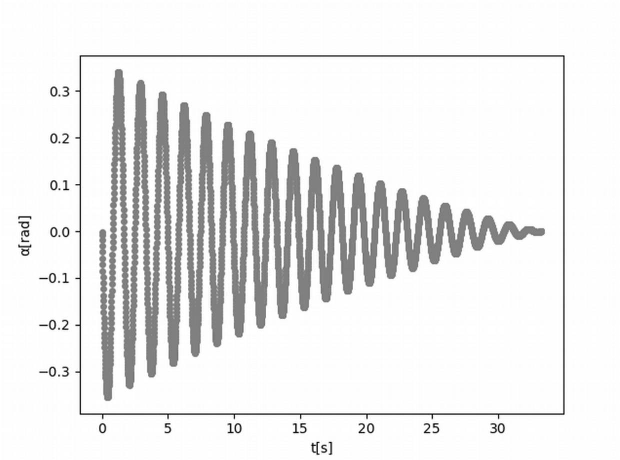

Figure 3 The recorded free vibrations of a pendulum

The points of the motion recorded within the initial 0.1s were used to identify the initial condition by fitting a linear function (Fig. 4).

It has been found that the initial angle of the motion from Fig. 3 is approximately equal to 0.0044 and the initial rotational velocity is close to -1.2693. Having the initial conditions identified, the standard error as a function of model parameters $\sigma = \sigma\mathbf{c}$) is well defined for each of the proposed friction models (5)–(7). Value of the standard error is estimated by solving the equation (3) with the identified initial conditions, by use of the appropriate friction force equation (5)–(7). The Runge-Kutta method implemented in the SciPy package of the Python 3 programming language has been applied to solve the equation of motion (3). The maximum integration step has been set to 10^{-3}. To optimize the function $\sigma(\mathbf{c})$, the "curve_fit" procedure of SciPy package has been used.

As the result, the following optimal parameters and corresponding standard error values have been obtained:

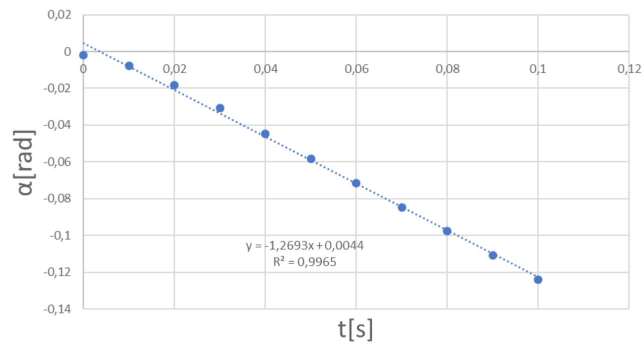

Figure 4 Identification of the initial conditions

1. Linear model: $c_1 \approx 0.1504$, $\sigma \approx 0.046$ (Fig. 5).
2. Coulomb model: $c_1 \approx 0.06151$, $c_2 \approx 0.04273$, $\sigma \approx 0.024$ (Fig. 6).
3. Cubic model: $c_1 \approx 0.2107$, $c_2 \approx -0.1116$, $\sigma \approx 0.024$ (Fig. 7).

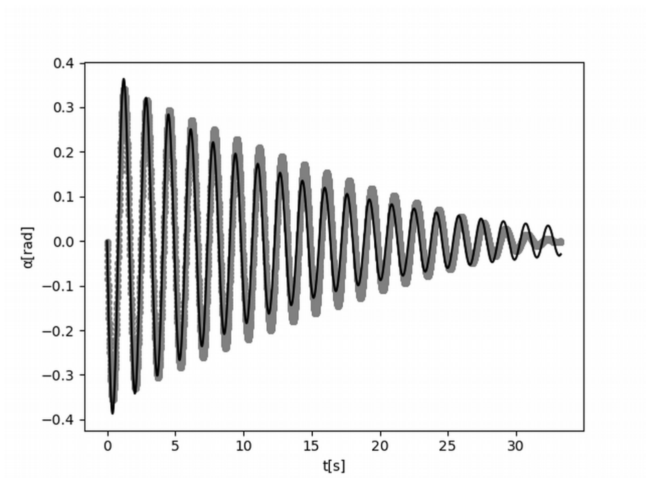

Figure 5 Comparison between the recorded and the simulated motion of the pendulum – linear friction model (5)

It can be noticed that application of the Coulomb model (6) and the cubic model (7) results in very similar accuracy of the model. However, there are important differences between them. The value of the second parameter c_2 of the cubic model is negative. Therefore, for large enough rotational velocities, the cubic model yields negative values of the friction force, which is obviously impossible in the real world. Therefore, the cubic model can be applied only in the limited range of velocities. On the other hand, the Coulomb model is not continuous. Due to that fact, the system cannot be linearized for the zero value of rotational velocity, which makes it difficult to determine stability of the control system (1)–(2) in the vicinity of an upright vertical position. Moreover, application of the Coulomb model may cause numerical problems during simulations.

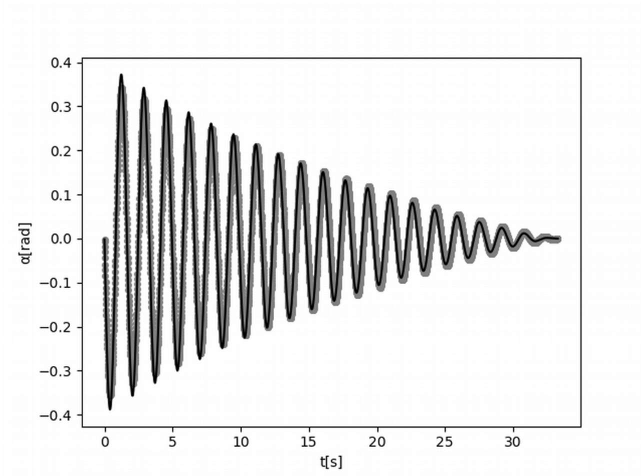

Figure 6 Comparison between the recorded and the simulated motion of the pendulum – Coulomb friction model (6)

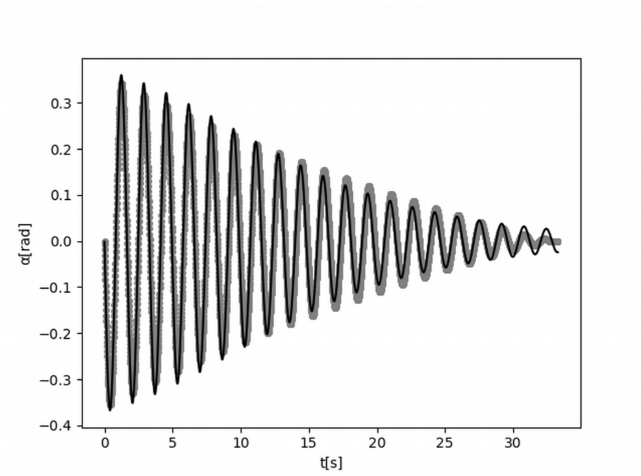

Figure 7 Comparison between the recorded and the simulated motion of the pendulum – cubic friction model (7)

4. Conclusions

This paper covers practical issues encountered during friction identification, which is necessary for modelling of the inverted pendulum. The control object to be modelled has been described. The method of identification based on free vibrations recording has been explained. Results of friction identification by means of three different models have been presented. The obtained outcomes have been discussed.

The performed experiment confirms that, in the particular application, the cubic friction model provides almost the same level of accuracy as the Coulomb friction model. Moreover, the cubic friction model is much more convenient in the case of control systems analysis due to the fact that it can be easily linearized. Therefore, stability of the system can be checked very easily when the cubic friction model is applied. On the other hand, it has been shown that a negative coefficient may be obtained in the cubic friction model, so it can be applied only in a specified range of velocities. Nevertheless, as long as the range of velocities is bounded and the ease of linearization is important, the cubic model can successfully compete with other solutions, such as Coulomb model.

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